CALCULATIONS INVOLVING SOLUTIONS

INTRODUCTION AND DEFINITIONS

Many chemical reactions take place in aqueous (water) solution. Quantities of such solutions are measured as volumes, while the amounts of solute present in a given volume of solution are the concentrations of the solutions.

The concentrations of solutions are commonly stated in moles (of solute) per litre (of solution) (mol. L⁻¹), or as Molarity (M), which means the same. In practical terms, however, amounts of solutes have to be measured as mass, for example in grams. It is not possible to measure the number of moles of a solid directly with a balance or any other instrument. Concentrations of solutions can also be expressed, therefore, in grams (of solute) per litre (of solution) (g. L⁻¹).

Grams per litre is therefore a practical way to express concentration of solute in a solution. Moles per litre, or molarity, is more important in making calculations and predictions about chemical reactions.

Volumes of solutions are defined in litres, (L), (which is sometimes expressed as dm³, or cubic decimetres¹). Millilitres (mL) are also used widely, but may have to be converted to litres for some calculations.

FORMULAE USEFUL FOR CALCULATIONS:

It is important, therefore, to be able to convert grams to moles and moles to grams for any substance.

\[
\text{Number of moles} = \frac{\text{Actual mass (grams)}}{\text{Molar Mass}} \quad \text{or} \quad n = \frac{m}{M}
\]

This can also be written:

\[
\text{Actual mass (grams)} = \text{Number of moles} \times \text{Molar mass}, \quad \text{or} \quad m = n \times M
\]

Concentration of a solution can be expressed as a formula:

\[
\text{Concentration (in mol. L}^{-1}\text{)} = \frac{\text{Number of moles of solute}}{\text{Volume of solution (litres)}} \quad \text{or} \quad C = \frac{n}{V}
\]

This can also be written:

\[
\text{Number of moles of solute} = \text{Concentration (mol. L}^{-1}\text{)} \times \text{Volume of solution (litres)}
\]

or \( n = C \times V \)

It is suggested strongly that these formulae be remembered as word formulae, rather than as algebraic formulae. With word formulae, the student is remembering the meanings of the parts of each formula. It is easy, with the algebraic formulae, to confuse n and m and M.

¹ One litre = 1 000 mL. Since 1.0 mL = 1.0 cm³, then 1.000 litre = 1000 cm³.

One decimetre (dm) = \( \frac{1}{10} \) metre = 10 cm.

\( (1.0 \text{ dm})^3 = (10 \text{ cm})^3 \).

1.0 dm³ = 1000 cm³ = 1.000 L.
Example one:
A solution of sodium hydroxide has a concentration = 0.50 mol. L⁻¹ (also written as 0.50M). How many grams of sodium hydroxide are dissolved in 1.000 L of solution?

Calculate first the number of moles in the 1.000 L of solution.

Concentration = 0.50M = \( \frac{\text{Number of moles of solute}}{\text{Volume of solution (litres)}} \) = \( \frac{\text{Number of moles of solute}}{1.000 \text{litre}} \)

Number of moles of solute = 0.50 x 1.000 = 0.50 mol
Sodium hydroxide: formula = NaOH, molar mass = (23.0 + 16.0 + 1.0) g = 40.0 g
One mole of NaOH = 40.0 g, so 0.50 mol of NaOH = 20.0 g.
20.0 g of solid sodium hydroxide dissolved in 1.000 L solution makes a solution of concentration = 0.50M.

Example two:
What mass of lead nitrate should be dissolved in 250 mL of water to make a solution of concentration 0.040M?

Lead nitrate: formula = Pb(NO₃)₂, molar mass = 331.2 g 250 mL = 0.250 L
Concentration = 0.040M = \( \frac{\text{Number of moles of solute}}{\text{Volume of solution (litres)}} \) = \( \frac{\text{Number of moles of solute}}{0.250 \text{litre}} \)

Number of moles of solute = 0.040 x 0.250 = 0.010 mol
Mass of lead nitrate required = (0.010 x 331.2) g = 3.31 g.

Example three:
What is the concentration of the solution if 10.0 g of sodium carbonate is dissolved in 200 mL of solution?

Sodium carbonate: formula = Na₂CO₃, molar mass = 106.0 g 200 mL = 0.200 L
Number of moles of Na₂CO₃ in 10.0 g = \( \frac{10.0}{106.0} \) = 0.0943 mol.
Concentration = \( \frac{\text{Number of moles of solute}}{\text{Volume of solution (litres)}} \) = \( \frac{0.0943}{0.200 \text{litre}} \) = 0.47 M

Example four:
What volume of solution must be made if 5.10 g of silver nitrate is to be dissolved to make a solution of concentration = 0.050M?

Silver nitrate: formula = AgNO₃, molar mass = 169.9 g
Number of moles of silver nitrate = \( \frac{\text{Actual mass (grams)}}{\text{Molar mass}} \) = \( \frac{5.10}{169.9} \) = 0.0300 mol
Concentration = \( \frac{\text{Number of moles of solute}}{\text{Volume of solution (litres)}} \) = \( \frac{0.050}{0.0300} \) = \( \frac{0.0300}{\text{Volume (litres)}} \)

Volume = \( \frac{0.0300}{0.050} \) L = 0.600 L = 600 mL

Calculations of this kind should be practised until they can be done quickly and easily.
Exercises:
1. How many moles of calcium hydroxide are needed to make 2.000 L of 0.001M solution? What mass of calcium hydroxide would be required?

2. What volume of 0.05M potassium sulfate contains 50.0 g of potassium sulfate? (Hint: first convert the number of grams to number of moles.)

3. What is the concentration of magnesium sulfate if 20.0 g of crystals (MgSO₄·7H₂O) are dissolved in 500 mL of solution?

4. What is the mass of hydrogen chloride in 2.50L of 10.5M hydrochloric acid solution? (Hydrochloric acid is a solution of hydrogen chloride gas in water. "Concentrated hydrochloric acid" is approximately 10.5M).

5. What mass of iron(II) sulfate is present in 25.00 mL of 0.10M solution?

Calculations Involving Particular Ions in a Solution
Consider a solution made by dissolving one mole of solid calcium chloride in one litre of water to make a solution that is 1.00M in calcium chloride.

\[
\text{CaCl}_2 (s) \rightarrow \text{Ca}^{2+}(aq) + 2\text{Cl}^-(aq)
\]

The one litre of solution contains 1.00 mol of CaCl₂. It can also be said to contain 1.00 mol of calcium ions and 2.00 mol of chloride ions.
This can be shown symbolically \([\text{CaCl}_2] = 1.00\text{M} \quad [\text{Ca}^{2+}] = 1.00\text{M} \quad [\text{Cl}^-] = 2.00\text{M}\)
(The square brackets, [ ], mean "concentration of ".)

Further Examples:
1. What is the concentration of ammonium ions in 0.25M ammonium sulfate solution?
   Formula for ammonium sulfate is \((\text{NH}_4)_2\text{SO}_4\). In a given volume of solution, there are twice as many moles of ammonium ions as there are moles of ammonium sulfate or of sulfate alone.
   \[
   [\text{NH}_4^+] = 2[\text{SO}_4^{2-}] = 2[(\text{NH}_4)_2\text{SO}_4]
   \]
   Therefore \([\text{NH}_4^+] = 0.50\text{M}\)

2. What is the concentration of nitrate ion in 0.40M aluminium nitrate solution?
   Formula for aluminium nitrate is \(\text{Al(NO}_3\text{)}_3\).
   Therefore \([\text{NO}_3^-] = 1.20\text{M}\)

3. What are the concentrations of iron(III) and of sulfate ions in 0.10M iron(III) sulfate?
   Formula for iron(III) sulfate is \(\text{Fe}_2(\text{SO}_4)_3\)
   Therefore \([\text{Fe}^{3+}] = 0.20\text{M} \quad \text{and } [\text{SO}_4^{2-}] = 0.30\text{M}\)

4. In a solution of silver sulfate in water, the concentration of silver ions is 0.0004M. What is the concentration of sulfate ions in the solution?
   Formula for silver sulfate is \(\text{Ag}_2\text{SO}_4\)
   \([\text{Ag}^+] = 2[\text{SO}_4^{2-}] \quad \text{Therefore } [\text{SO}_4^{2-}] = 0.0002\text{M}\)
Exercises:
1. What is the concentration of
   a) lead ions in a 0.02M solution of lead nitrate?
   b) nitrate ions in a 1.0M solution of zinc nitrate?
   c) ammonium ions in a 0.40M solution of ammonium chloride?
   d) iron(III) ions in a solution of iron(III) nitrate where \([\text{NO}_3^-] = 0.45\text{M}\)?
   e) iodide ions in a solution of magnesium iodide where \([\text{Mg}^{2+}] = 0.20\text{M}\)

2. What mass of solid would need to be dissolved in 250 mL of water to make a solution in which
   a) \([\text{Cl}^-] = 0.50\text{M}\), using potassium chloride?
   b) \([\text{SO}_4^{2-}] = 0.50\text{M}\), using sodium sulfate?
   c) \([\text{Fe}^{2+}] = 0.05\text{M}\), using iron(II) sulfate-7-water (\(\text{FeSO}_4\cdot7\text{H}_2\text{O}\))?
   d) \([\text{NH}_4^+] = 0.02\text{M}\), using ammonium hydrogenphosphate?
   e) \([\text{Cl}^-] = 0.60\text{M}\), using iron(III) chloride?

3. What is the molarity of chloride ions in each of the following solutions?
   a) 5.85 g of sodium chloride are dissolved in 500 mL of solution.
   b) 1.1 g of calcium chloride is dissolved in 100.0 mL of solution.
   c) 13.3 g of aluminium chloride dissolve in 600 mL of solution.
   d) 9.5 g of magnesium chloride and 0.74 g of potassium chloride are dissolved together in 250 mL of solution.
   e) 1.47 g each of zinc chloride, potassium chloride, and iron(III) chloride are dissolved together in 750 mL of solution.

4. To make a solution that is 0.10M in chloride, what volume of solution should be made
   a) if 10.7 g of ammonium chloride is dissolved to make the solution?
   b) if 22.2 g of calcium chloride is dissolved to make the solution?

CALCULATIONS INVOLVING CONCENTRATIONS OF SOLUTES IN CHEMICAL REACTIONS
The basic procedure here is similar to that for other calculations:
1) Write a balanced equation.
2) Write mole ratios under the equation, to show the ratio of moles of reactants to moles of products.
3) Write in the given data for concentrations and volumes, writing \(x\) for unknown values.
4) Convert values of concentration and volume to moles.
5) Use ratios to calculate \(x\). The ratio between the actual numbers of moles of the reactants (line 4 in example one on the next page) and the number of moles shown in the equation (line two in example one on the next page) are equal (line 5 in example one on the next page).

(Examples next page)
Example one:
What volume of 0.25M hydrochloric acid will exactly neutralise 40.0 mL of 2.00M sodium hydroxide solution?

1) Write a balanced equation: \( \text{HCl} + \text{NaOH} \rightarrow \text{NaCl} + \text{H}_2\text{O} \)

2) Write mole ratios under the equation.

3) Write in the given data for concentrations and volumes, writing \( x \) for unknown values.

4) Convert values of concentration and volume to moles.

5) Use ratios to calculate \( x \): 

\[
\frac{0.25x \times 1000\text{mol}}{1\text{mol}} = \frac{(2.00 \times 40.0) \times 1000\text{mol}}{1\text{mol}}
\]

The factor "/1000" converts the given volumes from millilitres to litres. Since it is a common factor, it is cancelled from the next step.

\[
x = \frac{2.00 \times 40.0}{0.25} = 320 \text{ mL}
\]

Example two:
Silver nitrate solution of concentration 0.025M is added to 50.0 mL of sodium sulfide solution, causing silver sulfide to precipitate. 21.5 mL of the silver nitrate solution was just sufficient to precipitate all the sulfide ions present. What was the concentration of sulfide ions in the sodium sulfide solution?

1) Write a balanced equation.
2) Write mole ratios under the equation.
3) Write in the given data for concentrations and volumes, writing \( x \) for unknown values.
4) Convert values of concentration and volume to moles.

When the factor 1000 has been cancelled, as in the previous example, the product of the concentration and volume in millilitres can be stated in millimoles (mmol).

\[
\frac{2 \text{AgNO}_3 \text{ (aq)} + \text{Na}_2\text{S (aq)}}{2 \text{ mol} \times 1 \text{ mol}} \rightarrow \frac{\text{Ag}_2\text{S (s)} + 2 \text{NaNO}_3 \text{ (aq)}}{(21.5 \times 0.025)\text{mmol} \times (50.0x)\text{mmol}}
\]

5) Use ratios to calculate \( x \). The ratios are \( \frac{\text{millimoles}}{\text{moles}} \). If the units used in both ratios are the same, the ratio can be calculated in the usual way.

\[
\frac{21.5 \times 0.025 \text{mmol}}{2 \text{ mol}} = \frac{50.0x \text{mmol}}{1 \text{ mol}}
\]

Molarity of \( \text{Na}_2\text{S} \) = \( \frac{21.5 \times 0.025}{2 \times 50.0} \) = 0.0054M = 5.4 \times 10^{-3}M

NB the concentrations of sodium ions and of nitrate ions in the mixture are equal.

\[
[\text{Na}^+] = 2 \times (\frac{50.0}{50.0 + 21.5}) \times 5.4 \times 10^{-3}M = 0.0075M, [\text{NO}_3^-] = (\frac{21.5}{50.0 + 21.5}) \times 0.025M = 0.0075M
\]
Example three:
When iron(II) sulfate solution is added to a solution of potassium cyanide, it reacts to form the hexacyanoferrate(II) ion, \(\text{Fe(CN)}_6^{4-}\). What is the largest volume of 0.50M iron(II) sulfate solution that would react completely with 120.0 mL of 0.40M KCN solution?

"Largest volume that would react" is the volume that is needed to react exactly. If a smaller volume were used, there would be excess of cyanide solution.

\[
\begin{align*}
\text{FeSO}_4 \text{ (aq)} &+ 6\text{KCN (aq)} \rightarrow \text{Fe(CN)}_6^{4-} \text{(aq)} + 6\text{K}^+ \text{(aq)} + \text{SO}_4^{2-} \text{(aq)} \\
1 \text{ mol} & \quad 6 \text{ mol} \\
0.50 \text{ M} & \quad 0.40 \text{ M} \\
x \text{ mL} & \quad 120.0 \text{ mL} \\
0.50x \text{ mmol} & \quad (0.40 \times 120.0) \text{ mmol} \\
\frac{0.50x \text{ mmol}}{1 \text{ mol}} & = \frac{0.40 \times 120.0 \text{ mmol}}{6 \times 0.50} = 16.0 \text{ mL}
\end{align*}
\]

Example four:
This example varies from the previous ones in that it involves an excess of one reactant.

30.0 mL of 0.25M sodium hydroxide solution is added to 12.5 mL of 0.40M sulfuric acid solution. What is the concentration of unreacted sodium hydroxide or sulfuric acid in the solution after the solutions are mixed?

\[
\begin{align*}
\text{2NaOH (aq)} + \text{H}_2\text{SO}_4(\text{aq}) & \rightarrow \text{Na}_2\text{SO}_4(\text{aq}) + 2\text{H}_2\text{O (l)} \\
2 \text{ mol} & \quad 1 \text{ mol} \\
0.25 \text{ M} & \quad 0.40 \text{ M} \\
30.0 \text{ mL} & \quad 12.5 \text{ mL} \\
(0.25 \times 30.0) \text{ mmol} & \quad (0.40 \times 12.5) \text{ mmol} \\
= 7.5 \text{ mmol} & = 5.0 \text{ mmol}
\end{align*}
\]

The equation shows that 2 mol of NaOH react with 1 mol of \(\text{H}_2\text{SO}_4\), so 7.5 mmol of NaOH require only 3.75 mmol of \(\text{H}_2\text{SO}_4\) for complete reaction. Sulfuric acid is in excess; the excess of \(\text{H}_2\text{SO}_4\) is \((5.0 - 3.75)\text{mmol} = 1.25 \text{ mmol}, or 1.25 \times 10^{-3} \text{ mol.}

Students uncomfortable with using millimoles can work with moles by inserting the factor 1000 to convert milliliters to liters wherever necessary: for example, the calculation above would appear as

\[
\begin{align*}
\text{2NaOH (aq)} + \text{H}_2\text{SO}_4(\text{aq}) & \rightarrow \text{Na}_2\text{SO}_4(\text{aq}) + 2\text{H}_2\text{O (l)} \\
2 \text{ mol} & \quad 1 \text{ mol} \\
0.25 \text{ M} & \quad 0.40 \text{ M} \\
30.0 \times 1000 \text{ mL} & \quad 12.5 \times 1000 \text{ mL} \\
(0.25 \times 30.0 \times 1000) \text{ mol} & = (0.40 \times 12.5 \times 1000) \text{ mol} \\
= 7.5 \times 10^{-3} \text{ mol} & = 5.0 \times 10^{-3} \text{ mol}
\end{align*}
\]

To calculate the concentration of \(\text{H}_2\text{SO}_4\) in the final solution: the number of moles of \(\text{H}_2\text{SO}_4\) is known, as is the volume of the solution: \((30.0 + 12.5) \text{ mL} = 42.5 \text{ mL} = 42.5 \times 10^{-3} \text{ L.}

The relationship \(\text{concentration} = \frac{\text{Number of moles of solute}}{\text{Volume of solution (litres)}}\) can be used:

\[
\text{Concentration of } \text{H}_2\text{SO}_4 = \frac{1.25 \times 10^{-3}}{42.5 \times 10^{-3}} = 0.024 \text{ M (Actually, HSO}_4^- \text{ ions are formed.)}
\]
Example five: *This is a more difficult problem.*

What volume of 0.10M silver nitrate solution should be added to 25.00 mL of 0.10M calcium chloride solution to leave a final chloride concentration = 0.05M?

The student must be aware of several things. Silver ions form insoluble AgCl when chloride is present. The initial concentration of calcium chloride is given as 0.10M, but the chloride concentration is 0.20M. It is inferred from the wording of the question that calcium chloride will be present in excess.

A chemical problem, however complex it may seem, should be started by writing a chemical equation and filling in the data under the equation in the way demonstrated previously. Unknown quantities, such as, in this case, the volume of silver nitrate solution, should be written as an algebraic symbol (such as x). Additional information may need to be inserted in the space under the equation.

In problems such as this, it is often easier to use an ionic equation than a molecular equation (see pages 23 - 24).

\[
\text{Ag}^+ \text{(aq)} + \text{Cl}^- \text{(aq)} \rightarrow \text{AgCl (s)}
\]

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<tr>
<td>1 mol</td>
<td>1 mol</td>
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<tr>
<td>0.10M</td>
<td>0.20M</td>
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<tr>
<td>x mL</td>
<td>25.00 mL</td>
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**AMOUNTS**

**IN MIXTURE**

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<tbody>
<tr>
<td>0.10x mmol</td>
<td>(0.20 x 25.00) mmol</td>
<td>5.00 mmol</td>
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**AMOUNTS**

**USED**

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<tr>
<td>0.10x mmol</td>
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**AMOUNT LEFT**

**AFTER REACTION**

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<tr>
<td>0 mmol</td>
<td>(5.00 - 0.10x) mmol</td>
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Total volume of solution after mixing = (25.00 + x) mL

Concentration of chloride, [Cl\^−], after mixing = \(\frac{(5.00-0.10x)}{(25.00+x)}\) = 0.05M

The equation in the previous line can be solved to give x = 23.0 mL

**Exercises:**

1. What volume of 0.12M potassium hydroxide will exactly neutralise 40.0 mL of 0.15M nitric acid?

2. What volume of 0.010M calcium hydroxide solution ("lime water") is neutralised exactly by 25.00 mL of 0.026M hydrochloric acid?

3. What is the concentration of lead ions in a solution if they are completely precipitated from 24.0 mL of solution by 32.0 mL of 0.28M potassium iodide. (Lead iodide is insoluble in cold water.)

4. What is the smallest concentration of sulfate ions that must be present for 22.0 mL of sodium sulfate solution to precipitate all the barium in 20.0 mL of 0.15M barium chloride solution? (Barium sulfate is insoluble in water.)
5. 25.0 mL of 0.065M sodium hydroxide solution is mixed with an equal volume of 0.030M sulfuric acid solution. What is the concentration of sodium hydroxide in the mixed solution?

6. Mercury(II) and iodide ions react to form an ion HgI$_4^{2-}$ (called tetraiodomercury(II)). What is the smallest volume of 0.40M potassium iodide solution that should be mixed with 10.0 mL of 0.10M mercury(II) chloride to convert all the mercury(II) ions to tetraiodomercury(II) ions?

7. Potassium dichromate solution is mixed with acidified sodium oxalate solution. The products of the reaction are chromium(III) ions and carbon dioxide. (See page 7 for formulae, page 28 or page 30 for methods of balancing equations.) 20.0 mL of 0.025M potassium dichromate solution was reduced completely by 30.0 mL of sodium oxalate solution. What is the concentration of chromium(III) in the final solution? *NB* there is a quick way to calculate the answer to this question.

8. Equal volumes of 0.020M sodium carbonate and 0.030M calcium nitrate solutions are mixed. What are the concentrations in the final mixture of a) sodium ions? b) nitrate ions? c) calcium ions? d) carbonate ions? *NB* calcium carbonate is insoluble in water.

9. If hydrogen peroxide solution is added to acidified potassium permanganate solution, the permanganate ions are reduced to Mn$^{2+}$ and oxygen gas is produced. What volume of 0.040M hydrogen peroxide will exactly reduce 20.00 mL of 0.0080M potassium permanganate solution? (See page 28 or page 30 for method to balance equation).

10. Excess solid zinc oxide is reacted with 40.0 mL of 1.0M hydrochloric acid until all of the acid has reacted. What is the concentration of zinc ions in the final solution? How many moles of zinc chloride will have been formed?

The following exercises are more difficult. Remember that even if you cannot see how to solve a problem, start by writing an equation (molecular or ionic - see page 23) and then entering mole ratios and given data under the equation, using algebraic symbols for them quantities that are unknown and have to be found.

11. Both lead chloride and lead bromide are insoluble in cold water. 20.0 mL of 0.12M sodium chloride solution are added to 50.0 mL of 0.05M lead nitrate solution. What volume of 0.20M potassium bromide solution would be needed to precipitate the remaining lead from the solution?

12. Barium sulfate and zinc hydroxide are both insoluble in water. Both substances will be precipitated if barium hydroxide solution and zinc sulfate solution are mixed. If 20.0 mL of barium hydroxide solution reacts exactly with 60 mL of zinc sulfate solution, what is the ratio of the concentrations of the two solutions?

13. Barium sulfate is insoluble in water. What volume of 0.10M barium hydroxide solution should be added to 20.0 mL of 0.15M sulfuric acid solution so that the concentration of sulfuric acid in the final solution is 0.10M?